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ALGORITHMIC AND PROGRAM SOFTWARE FOR COMBINATORIAL SIMULATION OF THE THERMAL REGIME OF SPACE OBJECTS

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The main content and basic principles of combinatorial mathematical simulation of the thermal regime for various structures and technical systems are revealed. Attention is paid to the fact that combinatorial simulation of the thermal regime requires the development of specialized algorithmic and program software, which admits flexible transformation of the parameters and structure of the initial mathematical model in the course of investigation. A brief analysis of the algorithms and programs developed is presented.

Space hardware as a whole is characterized by nonstandard approaches to arrangement and design. This is primarily explained by the very high functionality of spacecraft (S) and by the need to implement the required design characteristics under rigid mass, energy, and overall-dimension restrictions. Furthermore, it should be taken into account that a spacecraft must often operate under extreme conditions [1] (thermal, radiative, on exposure to vacuum, high pressures, aggressive media, etc.), the majority of spacecraft being thin-walled structures.

Elements forming the structure of a spacecraft have different functional purposes. However, proceeding from unified laws of design, these elements can be combined by a variety of general features. Among them are: the geometric features of structural elements (shells, including shells of revolution, plates, etc.); the methods for joining together structural elements of various dimensions; the presence and method of force stiffening of thin-walled structures; the presence of hydraulic heat-exchange main lines or heat pipes, etc. in the composition of the structure.

Usually, in investigating the thermal regime of a spacecraft, just as of any another technical system, there is an evolutionary process of determination and generalization of the regularities of its functioning, which consists in investigating the response of the structure considered to a change in external actions, to a change in the system parameters, the replacement of materials applied, etc. The evolutionary process of analyzing the thermal regime of the spacecraft can be implemented most successfully if, in forming mathematical thermal models and the corresponding algorithmic software, provision is made for their combinatorial change.

1. Basic Principles of Combinatorial Analysis of Thermal Regime. Let us consider the basic principles of combinatorial analysis [2] of the thermal regime of an unpressurized spacecraft. We will assume that in forming a mathematical model for the thermal regime, the definition domain of the problem is

$$D = D_{m2} + D_{m1} + D_{m0} + D_h + D_{h0} =$$
$$= \bigcup_{j_2=1}^{N_{m2}} D^{(j_2)} + \bigcup_{j_1=1}^{N_{m1}} D^{(j_1)} + \bigcup_{\alpha=1}^{N_{m0}} D^{(\alpha)} + \bigcup_{j_h=1}^{N_h} D^{(j_h)} + \bigcup_{\alpha_h=1}^{N_{h0}} D^{(\alpha_h)}, \tag{1}$$

in which we consider the set D_{m2} of two-dimensional distributed elements, the set D_{m1} of one-dimensional distributed elements, and the set D_{m0} of concentrated elements. We will also assume that a hydraulic channel

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consists of the set D_h of segments of pipelines or heat pipes and the set D_{h0} of hydraulic units, at which the separation or coalescence of the heat-transfer agent or thermal contact of the heat pipes occur.

Now we consider a generalized formulation of the problem of mathematical simulation of the thermal regime of the unpressurized spacecraft. Let, in the domain D, the models of the following thermophysical processes be determined:

$$L_{m2}^{(j_2)}(T(\vec{x}^{(j_2)})) = F_{m2}^{(j_2)}, \quad j_2 = \overline{1, N_{m2}}; \tag{2}$$

$$L_{m1}^{(j_1)}(T(\vec{x}^{(j_1)})) = F_{m1}^{(j_1)}, \quad j_1 = \overline{1, N_{m1}}; \tag{3}$$

$$L_{m0}^{(\alpha)}(T(x^{(\alpha)})) = F_{m0}^{(\alpha)}, \quad \alpha = \overline{1, N_{m0}}; \tag{4}$$

$$L_{h}^{(j_h)}(T(\vec{x}^{(j_h)})) = F_{h}^{(j_h)}, \quad j_h = \overline{1, N_h}; \tag{5}$$

$$L_{h0}^{(\alpha_h)}(T(x^{(\alpha_h)})) = F_{h0}^{(\alpha_h)}, \quad \alpha_h = \overline{1, N_{h0}}. \tag{6}$$

Here Eqs. (2) and (3) simulate heat conduction processes in the two-dimensional and one-dimensional distributed structural elements; Eqs. (4), the thermal state in the concentrated elements of the structure; Eqs. (5), the thermal regime of the heat-transfer agent in the elements of the thermal control system; Eqs. (6), the thermal state of the heat-transfer agent at the hydraulic units or thermal contact in the conjugation zone of the heat pipes.

For logical unification of individual models (2)-(6) within the framework of the generalized model, we introduce the relations of incidence for the model in the form of:

1) the joining conditions

$$\partial L_{m2}^{(j_2)-(j_2')}(T(\vec{x}^{(j_2)}), T(\vec{x}^{(j_2')})) = 0, \tag{7}$$

$$\partial L_{m2}^{(j_2)-(j_1)}(T(\vec{x}^{(j_2)}), T(\vec{x}^{(j_1)})) = 0, \tag{8}$$

$$\partial L_{m2}^{(j_2)-(\alpha)}(T(\vec{x}^{(j_2)}), T(x^{(\alpha)})) = 0, \tag{9}$$

$$\partial L_{m1}^{(j_1)-(j_1')}(T(\vec{x}^{(j_1)}), T(\vec{x}^{(j_1')})) = 0, \tag{10}$$

$$\partial L_{m1}^{(j_1)-(\alpha)}(T(\vec{x}^{(j_1)}), T(x^{(\alpha)})) = 0, \tag{11}$$

$$\partial L_{h}^{(j_h)-(j_h')}(T(\vec{x}^{(j_h)}), T(\vec{x}^{(j_h')})) = 0; \tag{12}$$

2) the balance relations

$$\sum_{j_2=1}^{N_{j_2,j_1}} \varphi_{m2}^{(j_2,j_1)} (T(\overrightarrow{x}^{(j_2)})) = \Phi_{m1}^{(j_2,j_1)} (T(\overrightarrow{x}^{(j_1)})) , \tag{13}$$

$$\sum_{j_1=1}^{N_{j_1,\alpha}} \varphi_{m1}^{(j_1,\alpha)} (T(\overrightarrow{x}^{(j_1)})) = \Phi_{m0}^{(j_1,\alpha)} (T(x^{(\alpha)})) , \tag{14}$$

$$\sum_{j_h=1}^{N_{h,\alpha_h}} \varphi_{h}^{(j_h,\alpha_h)} (T(\overrightarrow{x}^{(j_h)})) = \Phi_{h0}^{(j_h,\alpha_h)} (T(x^{(\alpha_h)})) . \tag{15}$$

In the course of combinatorial analysis of the thermal state of the spacecraft a change both in the parameters and in structure of the initial thermal model is possible. The content of the change in the model parameters can be seen from the following examples:

1) change in the models of the processes considered

$$L_{m2}^{(j_2)} (T(\overrightarrow{x}^{(j_2)})) = F_{m2}^{(j_2)} \rightarrow (L_{m2}^{(j_2)})'(T(\overrightarrow{x}^{(j_2)})) = (F_{m2}^{(j_2)})' ,$$

$$L_{m1}^{(j_1)} (T(\overrightarrow{x}^{(j_1)})) = F_{m1}^{(j_1)} \rightarrow (L_{m1}^{(j_1)})'(T(\overrightarrow{x}^{(j_1)})) = (F_{m1}^{(j_1)})' ,$$

$$L_{m0}^{(\alpha)} (T(x^{(\alpha)})) = F_{m0}^{(\alpha)} \rightarrow (L_{m0}^{(\alpha)})' (T(x^{(\alpha)})) = (F_{m0}^{(\alpha)})' ,$$

$$L_{h}^{(j_h)} (T(\overrightarrow{x}^{(j_h)})) = F_{h}^{(j_h)} \rightarrow (L_{h}^{(j_h)})' (T(\overrightarrow{x}^{(j_h)})) = (F_{h}^{(j_h)})' ,$$

$$L_{h0}^{(\alpha_h)} (T(x^{(\alpha_h)})) = F_{h0}^{(\alpha_h)} \rightarrow (L_{h0}^{(\alpha_h)})' (T(x^{(\alpha_h)})) = (F_{h0}^{(\alpha_h)})' ,$$

2) change in the joining conditions

$$\partial L_{m2}^{(j_2)-(j_2')} (T(\overrightarrow{x}^{(j_2)}), T(\overrightarrow{x}^{(j_2')})) = 0 \rightarrow (\partial L_{m2}^{(j_2)-(j_2')})' (T(\overrightarrow{x}^{(j_2)}), T(\overrightarrow{x}^{(j_2')})) = 0 ,$$

$$\partial L_{m2}^{(j_2)-(j_1)} (T(\overrightarrow{x}^{(j_2)}), T(\overrightarrow{x}^{(j_1)})) = 0 \rightarrow (\partial L_{m2}^{(j_2)-(j_1)})' (T(\overrightarrow{x}^{(j_2)}), T(\overrightarrow{x}^{(j_1)})) = 0 ,$$

$$\partial L_{m2}^{(j_2)-(\alpha)} (T(\overrightarrow{x}^{(j_2)}), T(x^{(\alpha)})) = 0 \rightarrow (\partial L_{m2}^{(j_2)-(\alpha)})' (T(\overrightarrow{x}^{(j_2)}), T(x^{(\alpha)})) = 0 ,$$

$$\partial L_{m1}^{(j_1)-(j_1')} (T(\overrightarrow{x}^{(j_1)}), T(\overrightarrow{x}^{(j_1')})) = 0 \rightarrow (\partial L_{m1}^{(j_1)-(j_1')})' (T(\overrightarrow{x}^{(j_1)}), T(\overrightarrow{x}^{(j_1')})) = 0 ,$$

$$\partial L_{m1}^{(j_1)-(\alpha)} (T(\overrightarrow{x}^{(j_1)}), T(x^{(\alpha)})) = 0 \rightarrow (\partial L_{m1}^{(j_1)-(\alpha)})' (T(\overrightarrow{x}^{(j_1)}), T(x^{(\alpha)})) = 0 ,$$

$$\partial L_{h}^{(j_h)-(j_h')} (T(\overrightarrow{x}^{(j_h)}), T(\overrightarrow{x}^{(j_h')})) = 0 \rightarrow (\partial L_{h}^{(j_h)-(j_h')})' (T(\overrightarrow{x}^{(j_h)}), T(\overrightarrow{x}^{(j_h')})) = 0 ;$$

3) change in the balance relations

$$\sum_{j_2=1}^{N_{j_2 j_1}} \varphi_{m2}^{(j_2 j_1)} (T(\vec{x}^{(j_2)})) = \Phi_{m1}^{(j_2 j_1)} (T(\vec{x}^{(j_1)})) \to$$

$$\to \sum_{j_2=1}^{N_{j_2 j_1}} (\varphi_{m2}^{(j_2 j_1)})' (T(\vec{x}^{(j_2)})) = (\Phi_{m1}^{(j_2 j_1)} (T(\vec{x}^{(j_1)})))',$$

$$\sum_{j_1=1}^{N_{j_1,\alpha}} \varphi_{m1}^{(j_1,\alpha)} (T(\vec{x}^{(j_1)})) = \Phi_{m0}^{(j_1,\alpha)} (T(x^{(\alpha)})) \to$$

$$\to \sum_{j_1=1}^{N_{j_1,\alpha}} (\varphi_{m1}^{(j_1,\alpha)})' (T(\vec{x}^{(j_1)})) = (\Phi_{m0}^{(j_1,\alpha)} (T(x^{(\alpha)})))',$$

$$\sum_{j_h=1}^{N_{j_h,\alpha_h}} \varphi_{h}^{(j_h,\alpha_h)} (T(\vec{x}^{(j_h)})) = \Phi_{h0}^{(j_h,\alpha_h)} (T^{(\alpha_h)}) \to$$

$$\to \sum_{j_h=1}^{N_{j_h,\alpha_h}} (\varphi_{h}^{(j_h,\alpha_h)})' (T(\vec{x}^{(j_h)})) = (\Phi_{h0}^{(j_h,\alpha_h)} (T^{(\alpha_h)}))';$$

4) change in the number of elements considered in the thermal model

$$N_{m2} \to (N_{m2})', \quad N_{m1} \to (N_{m1})', \quad N_{m0} \to (N_{m0})', \quad N_{h} \to (N_{h})', \quad N_{h0} \to (N_{h0})'.$$

Thus, in the course of the combinatorial investigation of the thermal regime there is a possibility not only to study the reference variant of the spacecraft, but also, as necessary, to refine flexibly the model, leaving its structure virtually unchanged. This process can be performed either by changing the model parameters, determined on the j_2th two-dimensional element, the j_1th one-dimensional element, and on the αth concentrated element; on the j_hth heat-transfer agent, the α_hth element of conjugation of the heat pipes or pipelines of the convective cooling system; or by means of the transformation of the incidence conditions and the balance relations, and also by refining the number of elements of the generalized model.

The second alternative of a change in the generalized model is the formation of a new structure of the thermal model that corresponds to the purposes of a specific investigation. In particular, this can be implemented by changing the dimensionality of the model:

$$L_{m2}^{(j_2)} (T(\vec{x}^{(j_2)})) = F_{m2}^{(j_2)} \to L_{m3}^{(j_2)} (T(\vec{x}^{(j_2)})) = F_{m3}^{(j_2)};$$

$$L_{m1}^{(j_1)} (T(\vec{x}^{(j_1)})) = F_{m1}^{(j_1)} \to L_{mn}^{(j_1)} (T(\vec{x}^{(j_1)})) = F_{mn}^{(j_1)}; \quad n = \overline{2, 3};$$

$$L_{m0}^{(\alpha)} (T(x^{(\alpha)})) = F_{m0}^{(\alpha)} \to L_{mn}^{(\alpha)} (T(\vec{x}^{(\alpha)})) = F_{mn}^{(\alpha)}; \quad n = \overline{1, 2, 3}.$$

It is obvious that the change in the model of only some j_2th two-dimensional element (Eqs. (2)) and of some j_1th element (Eqs. (3)) causes changes in the models of other elements (Eqs. (4)-(6)) and in the models of joining conditions (Eqs. (7)-(12)) and balance relations (Eqs. (13)-(15)), thus leading to a global change in the initial model (Eqs. (2)-(15)).

There will be a similar situation in the case where in solving the problem (Eqs. (2)-(15)), it is necessary to analyze in more detail, for example, the thermal state of an arbitrary concentrated element α. The thermal model of this element can be represented by the model of a higher level (Eqs. (2)-(4)), which increases naturally the dimensionality of the entire problem and also leads to the transformation of the initial generalized model. However, in this case there occurs in essence the incorporation of subcomplexes of the same type into the model complex (Eqs. (2)-(4)).

The presented basic principles of combinatorial analysis of the thermal regime can form a methodical basis for developing universal algorithmic and program software of the mathematical simulation and thermal design of the objects of space hardware that must ensure the solution of the following main problems: determination of a rational thermal regime of a spacecraft; investigation and selection of a regular variant for the support system of the thermal regime of a spacecraft; investigation of the influence of changes in the environmental action and of heat releases of the systems and units of a spacecraft on the thermal regime of spacecraft, study of the influence of a change in the design of a spacecraft, the replacement of materials applied, and a change in the characteristics and parameters of the support system of thermal regime and the parameters of radiative heat exchangers, contact resistances, etc. on the thermal state of the spacecraft; analysis of the thermal state of any structural element, each device, unit, system, etc.

2. Formation of the Combinatorial Mathematical Model. We consider this process using as an example the solution of the problem of mathematical simulation for the thermal regime of the spacecraft structure. Let us assume that, in the composition of the structure, it is possible to separate a certain set of two-dimensional thin-walled elements (plates, shells, etc.) determined in the region $D_{m2} = \bigcup_{j_2=1}^{N_{m2}} D^{(j_2)}$, the set of one-dimensional elements (ribs, rods, supports, frames, etc.) determined in the region $D_{m1} = \bigcup_{j_1=1}^{N_{m1}} D^{(j_1)}$, and the set of concentrated elements having a homogeneous temperature (the region $D_{m0} = \bigcup_{\alpha=1}^{N_{m0}} D^{(\alpha)}$).

Assuming that the problem under consideration is determined in the region

$$D = D_{m2} + D_{m1} + D_{m0} = \bigcup_{j_2=1}^{N_{m2}} D^{(j_2)} + \bigcup_{j_1=1}^{N_{m1}} D^{(j_1)} + \bigcup_{\alpha=1}^{N_{m0}} D^{(\alpha)}, \tag{16}$$

we present the mathematical model for the thermal regime of the structure in the following manner:

$$\forall\ \vec{x}^{(j_\delta)} \in D_{j_\delta},\ \ j_\delta = \overline{1, N_\delta},\ \ \bigcup_{j_\delta=1}^{N_\delta} D^{(j_\delta)} = D_{m\delta},\ \ \delta = 1, 2:$$

$$(1-\gamma_1^{(j_\delta)})\,\rho(\vec{x}^{(j_\delta)}, T)\, C_p(\vec{x}^{(j_\delta)}, T)\frac{\partial T}{\partial t} = \sum_{\beta=1}^{\delta} L_{x_\beta}^{(j_\delta)} T + (1-\gamma_2^{(j_\delta)})\, q_v(\vec{x}^{(j_\delta)}, T, t) + \frac{1}{V(\vec{x}^{(j_\delta)})}\times$$

$$\times \sum_{\beta=1}^{\delta} \sum_{\substack{j_\beta=1 \\ j_\beta\neq j_\delta}}^{N_{m\beta}} (1-\gamma_{3,s,b}^{(j_\delta,j_\beta)})\,\varepsilon(\vec{x}^{(j_\beta)})\,\lambda(\vec{x}^{(j_\beta)}, T)\, S_{s,b}^{(j_\delta,j_\beta)}(\vec{x}^{(j_\delta)}) \left.\frac{\partial T}{\partial \vec{x}^{(j_\beta)}}\right|_{\vec{x}^{(j_\beta)} \in (I^{(j_\beta)} \cap D^{(j_\delta)})} + \frac{1}{V(\vec{x}^{(j_\delta)})}\times$$

$$\times \sum_{\beta=1}^{\delta} \sum_{\substack{j_\beta=1 \\ j_\beta \neq j_\delta}}^{N_{m\beta}} (1-\gamma_{4,s,s}^{(j_\delta,j_\beta)})\, k_{s,s}^{(j_\delta,j_\beta)} (T(\vec{x}^{(j_\beta)}) - T(\vec{x}^{(i_\delta)}))\, S_{s,s}^{(j_\delta,j_\beta)} (\vec{x}^{(j_\delta)})\,|_{\vec{x}^{(j_\beta)} \in (D^{(j_\beta)} \cap D^{(j_\delta)})} +$$

$$+ \frac{1}{V(\vec{x}^{(j_\delta)})} \sum_{\alpha=1}^{N_{m0}} (1-\gamma_{5,s,\alpha}^{(j_\delta,\alpha)})\, k_{s,\alpha}^{(j_\delta,\alpha)} (T(x^{(\alpha)}) - T(\vec{x}^{(i_\delta)}))\, S_{s,\alpha}^{(j_\delta,\alpha)} (\vec{x}^{(j_\delta)})\,|_{x^{(\alpha)} \in (D^{(\alpha)} \cap D^{(j_\delta)})}\,; \tag{17}$$

$$L_{x_\beta}^{(j_\delta)} = \frac{\partial}{\partial x_{x_\beta}^{(j_\delta)}} \left(\lambda (x_{x_\beta}^{(j_\delta)})\, S(x_{x_\beta}^{(j_\delta)}) \frac{\partial}{\partial x_{x_\beta}^{(j_\delta)}} \right) \frac{1}{S(x_{x_\beta}^{(j_\delta)})}\,; \tag{18}$$

$$q_v(\vec{x}^{(j_\delta)}, T, t) = \sum_{i_{hr}=1}^{N_{hr}^{(j_\delta)}} (1-\beta_{hr}^{(i_{hr})}(\vec{x}^{(j_\delta)}))\, q_{hr}^{(i_{hr})}(\vec{x}^{(j_\delta)}, T, t) +$$

$$+ \frac{1}{V(\vec{x}^{(j_\delta)})} \sum_{i_s=1}^{N_s^{(j_\delta)}} (1-\beta_{s,v}^{(i_s)}(\vec{x}^{(j_\delta)})) \cdot q_{s,v}^{(i_s)}(\vec{x}^{(j_\delta)}, T, t) \cdot S_{s,v}^{(i_s)}(\vec{x}^{(j_\delta)})\,; \tag{19}$$

$$T(\vec{x}^{(j_\delta)})\,|_{t=0} = T_0(\vec{x}^{(j_\delta)})\,; \tag{20}$$

$$T(\vec{x}^{(j_\delta)})\,|_{\vec{x}^{(j_\delta)} \in \Gamma^{(i_\delta)}} = T_b(\vec{x}^{(j_\delta)}) \tag{21}$$

or

$$\varepsilon(\vec{x}^{(j_\delta)})\, \lambda(\vec{x}^{(j_\delta)}, T) \left.\frac{\partial T}{\partial \vec{x}^{(j_\delta)}}\right|_{\vec{x}^{(j_\delta)} \in \Gamma^{(j_\delta)}} = q_b^{(j_\delta)}(\vec{x}^{(j_\delta)}, T, t)\,; \tag{22}$$

$$\varepsilon(\vec{x}^{(j_\delta)}) = \begin{cases} -1\,, & \vec{x}^{(j_\delta)} = (\vec{x_0})^{(j_\delta)}\,, \\ +1\,, & \vec{x}^{(j_\delta)} = (\vec{x}_{fin})^{(j_\delta)}\,; \end{cases} \tag{23}$$

$$q_b^{(j_\delta)}(\vec{x}^{(j_\delta)}, T, t) = \sum_{i_b=1}^{N_b^{(j_\delta)}} (1-\beta_b^{(j_\delta)})\, q_b^{(j_\delta)}(\vec{x}^{(j_\delta)}, T, t)\,|_{\vec{x}^{(j_\delta)} \in \Gamma^{(j_\delta)}} +$$

$$+ \sum_{\beta=1}^{\delta} \sum_{\substack{j_\beta=1 \\ j_\beta \neq j_\delta}}^{N_{b\beta}} (1-\beta_{b,b}^{(j_\delta,j_\beta)})\, \varepsilon(\vec{x}^{(j_\beta)})\, \lambda(\vec{x}^{(j_\beta)}, T)\, S_{b,b}^{(j_\delta,j_\beta)}(\vec{x}^{(j_\delta)}) \left.\frac{\partial T}{\partial \vec{x}^{(j_\beta)}}\right|_{\vec{x}^{(j_\beta)} \in (\Gamma^{(j_\beta)} \cap \Gamma^{(j_\delta)})} +$$

$$+\sum_{\beta=1}^{\delta}\sum_{\substack{j_\beta=1\\ j_\beta\neq j_\delta}}^{N_{s\beta}} (1-\beta_{b,s}^{(j_\delta,j_\beta)})\, k_{b,s}^{(j_\delta,j_\beta)}\,(\overrightarrow{x}^{(j_\delta)})\,(T(\overrightarrow{x}^{(j_\beta)})-T(\overrightarrow{x}^{(i_\delta)}))\, S_{b,s}^{(j_\delta,j_\beta)}\,(\overrightarrow{x}^{(j_\delta)})|_{\overrightarrow{x}^{(j_\beta)}\in(D^{(j_\beta)}\cap\Gamma^{(j_\delta)})}+$$

$$+\sum_{\alpha=1}^{N_{m0}} (1-\beta_{b,\alpha}^{(j_\delta,\alpha)})\, k_{b,\alpha}^{(j_\delta,\alpha)}\,(\overrightarrow{x}^{(j_\delta)})\,(T(x^{(\alpha)})-T(\overrightarrow{x}^{(i_\delta)}))\, S_{b,\alpha}^{(j_\delta,\alpha)}\,(\overrightarrow{x}^{(j_\delta)})|_{\overrightarrow{x}^{(j_\delta)}\in(\Gamma^{(j_\delta)}\cap D^{(\alpha)})}; \tag{24}$$

$$\forall\, \overrightarrow{x}^{(\alpha)}\in D^{(\alpha)},\quad \alpha=\overline{1,N_{s0}}:$$

$$(1-\delta_1^{(\alpha)})\, C_{m_\alpha}\frac{\partial T(x^{(\alpha)})}{\partial t}=(1-\delta_2^{(\alpha)})\, Q_v^{(\alpha)}+$$

$$+(1-\delta_{3,\alpha,b}^{(\alpha,i_1)})\sum_{j_1=1}^{N_{m1}}\varepsilon(\overrightarrow{x}^{(j_i)})\,\lambda(\overrightarrow{x}^{(j_i)},T)\, S_{\alpha,b}^{(\alpha,j_1)}\,(\overrightarrow{x}^{(j_i)})\left.\frac{\partial T}{\partial x^{(j_1)}}\right|_{\overrightarrow{x}^{(j_1)}\in(\Gamma^{(j_1)}\cap D^{(\alpha)})}+$$

$$+\sum_{j_2=1}^{N_{m2}}(1-\delta_{4,\alpha,b}^{(\alpha,i_2)})\,\varepsilon(\overrightarrow{x}^{(j_2)})\,\lambda(\overrightarrow{x}^{(j_2)},T)\, S_{\alpha,b}^{(\alpha,j_2)}\left.\frac{\partial T}{\partial\overrightarrow{x}^{(j_1)}}\right|_{\overrightarrow{x}^{(j_2)}\in(\Gamma^{(j_2)}\cap D^{(\alpha)})}+$$

$$+\sum_{\beta=1}^{\delta}\sum_{j_\beta=1}^{N_{m\beta}}(1-\delta_{5,\alpha,s}^{(\alpha,j_\beta)})\, k_{\alpha,s}^{(\alpha,j_\beta)}\,(x^{(\alpha)})\,(T(\overrightarrow{x}^{(j_\beta)})-T(x^{(\alpha)}))\, S_{\alpha,s}^{(\alpha,j_\beta)}\,(x^{(\alpha)})|_{\overrightarrow{x}^{(j_\beta)}\in(D^{(j_\beta)}\cap D^{(\alpha)})}+$$

$$+\sum_{\substack{\alpha'=1\\ \alpha\neq\alpha'}}^{N_{m0}}(1-\delta_{6,\alpha,s}^{(\alpha,\alpha')})\, k_{\alpha,s}^{(\alpha,\alpha')}\,(x^{(\alpha')})\, T(x^{(\alpha')})\,(T(x^{(\alpha)}))\, S_{\alpha,s}^{(\alpha,\alpha')}\,(x^{(\alpha)})|_{x^{(\alpha)}\in(D^{(\alpha)}\cap D^{(\alpha')})}; \tag{25}$$

$$Q_v^{(\alpha)}=(1-\beta_1^{(\alpha)})\, Q_{hr}^{(\alpha)}+(1-\beta_2^{(\alpha)})\int_{S_{ext}^{(\alpha)}} q_{ext}^{(\alpha)}\, dS_{ext}^{(\alpha)}+$$

$$+(1-\beta_3^{(\alpha)})\int_{S_{int}^{(\alpha)}} q_{int}^{(\alpha)}\, dS_{int}^{(\alpha)}+(1-\beta_4^{(\alpha)})\,\alpha_{conv}\,(T(x^{(j_h)})-T(x^{(\alpha)}))\, S_{conv}^{(\alpha)}; \tag{26}$$

$$\varepsilon(x^{(j_1)})=\begin{cases}-1, & x^{(j_1)}=(x_0)^{(j_1)};\\ +1, & x^{(j_1)}=(x_{fin})^{(j_1)};\end{cases} \tag{27}$$

$$T(x^{(\alpha)})|_{t=0}=T_0(x^{(\alpha)}). \tag{28}$$

The considered system of equations above is a rather universal statement of the problem of mathematical simulation the thermal regime of three-dimensional structures. Let us reveal some features of representation of the equations that enter into this system.

Equation (16) describes the temperature distribution in each j_δth distributed structural element $j_\delta = \overline{1, N_\delta}$, $\delta = 1, 2$. The control δ-functions $\gamma_1^{(j_\delta)}$, $\gamma_2^{(j_\delta)}$, $\gamma_{3,s,b}^{(j_\delta,\gamma_\beta)}$, $\gamma_{4,s,s}^{(j_\delta,j_\beta)}$, and $\gamma_{5,s,\alpha}^{(j_\delta,\alpha)}$ used here have the following values:

$$\gamma_{i,\dots}^{(j_\delta,\dots)} = \begin{cases} 0, & \text{if } i = m, \\ 1, & \text{if } i \neq m, \end{cases}$$

where m is the number of the corresponding term in Eq. (17). Thus, these control δ-functions ensure the flexible formation and modification of the thermal model for the distributed structural elements.

Of similar meaning are the control δ-functions $\delta_1^{(\alpha)}$, $\delta_2^{(\alpha)}$, $\delta_{3,\alpha,b}^{(\alpha,j_1)}$, $\delta_{4,\alpha,b}^{(\alpha,j_2)}$, $\delta_{5,\alpha,s}^{(\alpha,j_\beta)}$, and $\delta_{6,\alpha,s}^{(\alpha,\alpha')}$ in Eq. (25) for the thermal state of the concentrated elements and also in the δ-functions $\beta_{hr}^{(j_{hr})}(\vec{x}^{(j_\delta)})$ and $\beta_{s,y}^{(j_s)}(\vec{x}^{(j_\delta)})$ in expression (19) determining the heat source in Eq. (17), in the functions $\beta_b^{(j_\delta)}$, $\beta_{b,b}^{(j_\delta,j_\beta)}$, $\beta_{b,s}^{(j_\delta,j_\beta)}$, and $\beta_{b,\alpha}^{(j_\delta,\alpha)}$ in expression (24) for the boundary conditions of Eq. (17), and $\beta_i^{(\alpha)}$, $i = \overline{1, 4}$, in expression (26) that reveal the structure of operating heat releases in Eq. (25).

In the present work, we will simulate the support system of the thermal regime of the spacecraft by assigning the corresponding components in the heat-source functions and in the expressions for the boundary conditions of the equations under consideration. It is assumed that each component of the heat-source function simulates the operating thermophysical process in criterial form. As necessary, to determine the refined transfer coefficients, we can use in addition the corresponding program software.

It should also be noted that Eqs. (17) and (25) simulate, apart from the thermal state of the distributed and concentrated elements, the boundary and contact thermal interaction between the structural elements. Furthermore, determination of the elements of the boundary conditions in the form of Eq. (22) at the external boundaries enables one to change flexibly the structure of the boundary thermal interaction.

Analysis of the aforesaid formulation of the problem of mathematical simulation of the thermal regime for spatial structures indicates that this system of equations can be flexibly transformed in order to allow for the features of a specific design. This is achieved by selecting a certain combination of the equations considered, by selecting the required structure of each equation, and by constructing the corresponding thermal coupling system, which is mapped in the boundary and thermal-conjugation conditions.

Thus, the mathematical model considered for the thermal regime of the spacecraft structure can be used in combinatorial analysis of the thermal regime of space objects, since in the course of investigations it allows one to change the parameters and structure of the model.

As indicated above, the combinatorial change in model parameters is usually implemented through the change in the thermophysical characteristics, the transfer coefficients, the source function in Eqs. (16)-(28), and also on changing the thermal-conjugation conditions (incidence conditions) and in the case of change in the number of the model elements.

In practice this means that a researcher has a chance not only to study the thermal state of the basic and reference variant of the given structure of the spacecraft but also, if necessary, by refining flexibly the model without changing simultaneously its structure, to determine the thermal state of modifications of the considered structure of the spacecraft.

The second direction of the combinatorial change in the mathematical thermal model is its structural change. When the thermal-model structure is changed both the "simplification" and "complication" of the problem can be used, which is associated both with the necessity of a more comprehensive representation of operating conditions for the structure and with restrictions that can be imposed by the computaters used.

The "simplification" of the problem can consist, for example, in determining the mean-integral thermal state of the structural elements, when it is possible to use models with concentrated parameters. Under the conditions of orbital flight, a similar approach allows one to determine with a sufficient degree of accuracy the thermal state of the structure and systems of automatic spacecraft. In designing a descent spacecraft (DS) intended for returning a payload to the earth, researchers are often oriented to the investigation of a one-dimen-

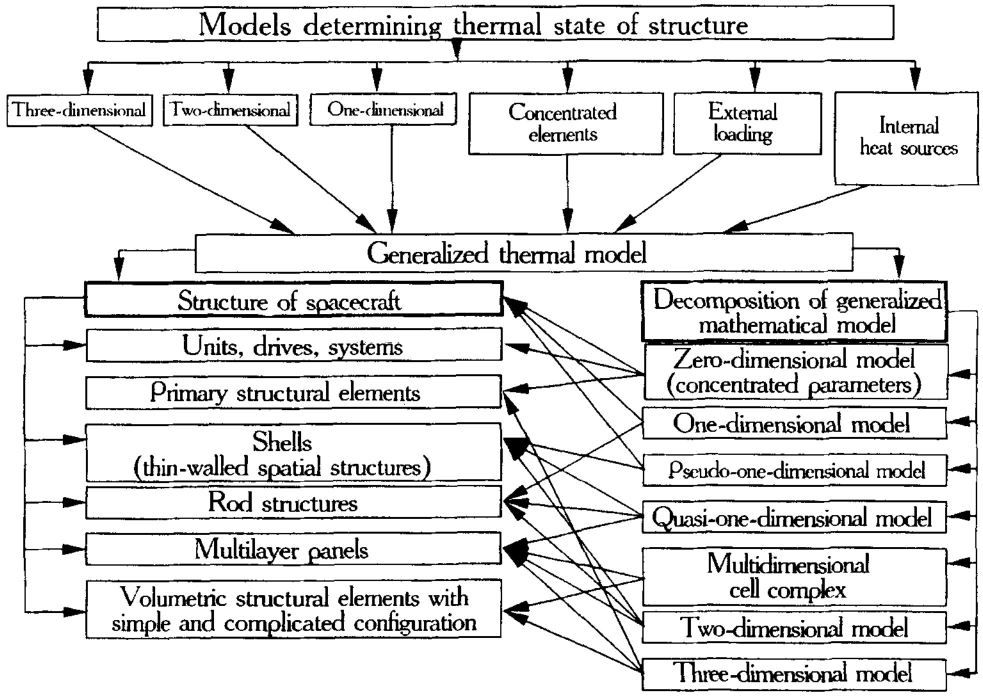

Fig. 1. Structure of a combination of mathematical models for the thermal regime of spacecraft structures.

sional process of heating and destruction of its thermal protection at individual points. It should be noted that, in this case, use is made of the model with distributed parameters.

Under the conditions of multiple-factor spatial thermal loading, the "complete" formulation of the initial problem is usually used. If necessary, its "simplification" is conducted by decreasing the dimensionality of the models of distributed elements or by replacing distributed elements by a certain set of concentrated elements or subelements of lower dimensionality. In the latter case, use can be made of quasi-one-dimensional models and models based on a multidimensional CW-complex, etc.

In turn, the "complication" of the formulation of the initial problem is implemented by means of increasing the number of elements, included in the mathematical model for the thermal regime of the structure considered, by "increasing" the dimensionality of distributed elements, and by complicating thermal couplings due to the application of more complete mathematical models of the external thermal actions and heat transfer between the elements of a given structure and models of the processes in systems and units.

3. Algorithmic and Program Software of Combinatorial Simulation of Thermal Regime for Spacecraft. Thus, the generalized formulation of the problem analyzed can be considered as basic for implementing the combinatorial simulation of the thermal regime for space structures. Within the framework of this formulation in the course of investigation one can use both the models with concentrated parameters and models with distributed parameters of the corresponding dimensionality and any combination of them. In addition, it is possible to include into the number of the models with distributed parameters pseudo-one-dimensional design models for the thermal regime of spacecraft, quasi-one-dimensional models for two-dimensional shell structures, a multidimensional CW-complex for three-dimensional structural elements, etc. The structure of the considered combination of mathematical models is presented in Fig. 1.

The algorithmic implementation of this combination of mathematical models is oriented to the finite-difference method. In the finite-difference approximation of differential operators and the formation of the grid function of heat sources, each node of the difference grid, according to the integro-interpolation method, is made to correspond to the volume element [2]. The values of the variables and the functions investigated at these nodes are determined as mean-integral ones within the limits of the volume element. In addition, each volume element is made to correspond to the area elements, within the limits of which the mean-integral thermal action of the heat sources is considered and thereby their nodal values are determined.

The algorithm based on a modified method of "skeleton" structures [3] is used as a basic one. For simulating the thermal regime of individual shell structures, multilayer panels, etc., use is made of the traditional variable-direction method and locally one-dimensional schemes [2]. To calculate the thermal regime of the structures in a one-dimensional approximation (involving pseudo-one-dimensional and quasi-one-dimensional formulations, and also a multidimensional CW-complex [4]), the algorithm oriented to the graph of the thermal model of a general form [5] is used as the most universal one. In simulating the thermal regime of the spacecraft structure in a zero-dimensional formulation, iteration methods for solving algebraic systems are applied.

The developed program software is oriented to the employment of present-day personal computers and is intended to solve research and design problems. The program software is constructed so as to enable one, in the combinatorial formulation, to investigate the thermal regime of both the individual systems and elements of the spacecraft structure and the spacecraft as a whole. The GRAPH, BRIZ, and ELEMENT systems are the basic elements of the program software.

The GRAPH software system ensures combinatorial analysis of the thermal regime for truss and rod structures of unpressurized spacecraft and makes it possible to calculate the temperature fields in spatial elements of the spacecraft structure in pseudo-one-dimensional and quasi-one-dimensional formulations.

The BRIZ software system determines temperature fields and operating thermal loading in the spacecraft structures that share the "axial" or "peripheral" coordinates. They can include radiative heat exchangers, elements of the spacecraft body, antennae, fuel sections, etc. This system supports one of the versions of the PANEL software system intended to investigate the thermal state of heat-exchange panels of unpressurized spacecraft. The ELEMENT software system synthesizes the possibilities of the GRAPH and BRIZ systems and allows one to investigate the thermal state of unpressurized spacecraft of any design. In the course of investigation not only the temperature fields in the structure are calculated but also the thermal state of radioelectronic equipment and various systems and units of the spacecraft are determined as well as the parameters of the environmental thermal action.

NOTATION

D, domain of definition of the problem; D_{m2}, spatial domain that corresponds to the two-dimensional element; D_{m1}, the same, to the one-dimensional element; D_{m0}, the same, to the concentrated structural element; D_h, domain of definition of the element of the convective thermal control system (segment of a pipeline or a heat pipe); D_{h0}, domain of definition of the hydraulic units; Γ, boundary of domain D; L, operator of the mathematical model of the thermophysical process; ∂L, operator of joining conditions; φ, operator of balance relations; L_x, Laplace operator for the parabolic-type equation; F, right-side function of the mathematical model of the thermophysical process; Φ, right-side function of balance relations; q_v, function of the volumetric heat source; q_{hr} and q_s, components of the volumetric and surface heat sources in the heat-source function for the distributed element; Q_v, volumetric heat source operating in the concentrated element; Q_{hr}, volumetric heat releases in the concentrated element; q_b, q_{ext}, q_{int}, densities of boundary, external, and internal heat fluxes, respectively; β, δ, γ, δ-functions that determine the appearance of the thermal model; T, temperature; λ, thermal conductivity; ρ, density; C_p, heat capacity at constant pressure; $C_m = C_p m$, mass heat capacity; m, mass; α_{conv}, coefficient of convective heat exchange; k, coefficient of surface heat transfer between structural elements; t, time; x, $\vec{x}$, spatial coordinate; S, area; V, volume; N, number of elements; N_{m2}, number of two-dimensional elements; N_{m1}, the same, of one-dimensional elements; N_{m0}, the same, of concentrated structural elements; N_h, number of elements of the convective thermal control system (segments of pipelines or heat pipes); N_{h0}, number of hydraulic and contact units of the thermal control system. Superscripts and subscripts: j, distributed structural element; m1, one-dimensional structural element; m2, two-dimensional structural element; α, concentrated structural element; h, heat carrier; α_h, conjugation element of heat pipes or pipelines of the convective cooling system; β, δ, direction for multidimensional structural elements; b, boundary; s, surface; int, internal heat factors; ext, external heat factors; conv, convective heat transfer; v, volumetric heat source; hr, heat release; s, v, surface component of the volumetric source; b,b, boundary thermal interaction between the distributed elements; b,s, boundary thermal interaction with the surface of the distributed elements; b,α, boundary thermal interaction with the surface of the concentrated elements; s,b, surface thermal interaction with the boundaries of the distributed elements; s,s, surface thermal interaction of the distributed elements; s,α, surface thermal interaction with the surface of the concentrated elements; α,b, thermal interaction of the concentrated elements with the boundaries of the distributed elements; α,s, surface thermal interaction of the concentrated and distributed elements; α,α, surface thermal interaction of the concentrated elements; 0, initial instant of time, initial coordinate; fin, finite instant of time, finite coordinate.

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